EXERCISE – II HINTS & SOLUTIONS

Sol.1 A,B

$(1 + x^2)^2 (1 + x)^n = A_0 + A_1x + A_2x^2 \ldots\ldots$

If A_0, A_1, A_2 are in A.P.

$\Rightarrow (1 + x^4 + 2x^2)(1 + x)^n = A_0 + A_1x + A_2x^2 \ldots\ldots$

If A_0, A_1, A_2 are in A.P.

$$\Rightarrow (1+x^4+2x^2)\left[1+nx+\frac{n(n-1)x^2}{2!}+\frac{n(n-1)(n-2)}{3!}x^3+\ldots\ldots\right] = A_0 + A_1x + A_2x^2 + \ldots.$$

by comparism

$A_0 = 1$

$A_1 = n \quad \& A_2 = \frac{n(n-1)}{2!}+2=\frac{n^2-n+4}{2}$

$2A_1 = A_0 + A_2$

$\Rightarrow 2n = 1 + \frac{n^2-n+4}{2}$

$\Rightarrow 4n = 2 + n^2 - n + 4$

$\Rightarrow n^2 - 5n + 6 = 0$

$\Rightarrow (n - 2)(n - 3) = 0$

$\Rightarrow n = 2 \quad$ or $\quad n = 3$

Sol.2 A,B,C

$101^{100} - 1 = (1 + 100)^{100} - 1$

$= {}^{100}C_0 + {}^{100}C_1(100)^1 + {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}(100)^{100} - 1$

$= 100 \times 100 + {}^{100}C_2(100)^2 + \ldots\ldots + {}^{100}C_{100}\,100^{100}$

$= 10000\,[1+{}^{100}C_2+{}^{100}C_3(100)+\ldots+{}^{100}C_{100}\,100^{98}]$

$= 10000$ [Integer]

which is divisible by 100, 1000, 10000

Sol.3 B,C,D

6^{th} term in the Expension of $\left[\frac{3}{2}+\frac{x}{3}\right]^n$ for x = 3

is numerically greatest $\frac{n+1}{\left|\frac{x}{y}\right|+1}-1 \leqslant r \leqslant \frac{n+1}{\left|\frac{x}{y}\right|+1}$

$\Rightarrow \frac{n+1}{\frac{3}{2}+1}-1 \leqslant 5 \leqslant \frac{n+1}{\frac{3}{2}+1}$

$\Rightarrow \frac{2(n+1)}{5}-1 \le 5 \le \frac{2(n+1)}{5}$

$\Rightarrow \frac{2(n+1)}{5} \leqslant 6$ and $\frac{2(n+1)}{5} \geqslant 5$

$\Rightarrow n + 1 \leqslant 15$ and $n+1 \geqslant \frac{25}{2}$

$\Rightarrow n \leqslant 14$ and $n+1 \geqslant \frac{23}{2}$

$\Rightarrow 11.4 \leqslant n \leqslant 14$

$n \in N \Rightarrow n = 12, 13, 14$

for these value of n 6^{th} term is greatest term

Sol.4 A,C,D

$I + f = (9+\sqrt{80})^n$, $0 < f < 1$, I & $n \in N$

Let $f' = (9-\sqrt{80})^n$, $0 < f' < 1$

$I + f + f' = 2$ [Integer] = Even Integer

$\Rightarrow (f + f')$ should be integer

$\because 0 < f + f' < 2$

$\Rightarrow f + f' = 1 \Rightarrow f' = 1 - f$

$= (9-\sqrt{80})^n \Rightarrow$ (D)

$I + f + f' = I + 1$ = even integer

$\Rightarrow I$ = odd integer $\Rightarrow$ (A)

$(I + f)(1 - f) = (9+\sqrt{80})^n (9-\sqrt{80})^n = (81 - 80)^n$

$= 1^n = 1 \Rightarrow$ (C)

Sol.5 **B,C,D**

$\left(x^{2/3} - \frac{1}{\sqrt{x}}\right)^{30}$ term x^{13}

$T_{r+1} = {}^{30}C_r x^{\frac{2}{3}(30-r)} (+x)^{-\frac{1}{2}r} (-1)^r = {}^{30}C_r (-1)^r x^{\frac{120-7r}{6}}$

$\frac{120-7r}{6} = 13 \Rightarrow \frac{120-78}{7} = r \Rightarrow r = 6$

$= {}^{30}C_6 = \frac{30\cdot 29\cdot 28\cdot 27\cdot 26\cdot 25}{6\cdot 5\cdot 4\cdot 3\cdot 2}$

which is divisibe by 29, 63, 65

Sol.6 **B,C,D**

$\left(x^3 + 3.2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11} = \left(x^3 + \frac{3}{x^3}\right)^{11}$

$T_{r+1} = {}^{11}C_r (x^3)^{11-r}\ 3^r (x^3)^{-r} = {}^{11}C_r\ 3^r\ (x^3)^{11-2r}$

$= {}^{11}C_r\ 3^r\ (x^3)^{11-2r} = {}^{11}C_r\ 3^r\ x^{33-6r}$

(A) $33 - 6r = 2 \Rightarrow \frac{31}{6} = r \Rightarrow$ Not possible

(B) x^2 doesn't appear

(C) $33 - 6r = -3 \Rightarrow 36 = 6r \Rightarrow r = 6$

(x^{-3}) term exist/appear in exp.

(D) for x^3, r = 5, & x^{-3}, r = 6

$\frac{T_6}{T_7} = \frac{{}^{11}C_5\ 3^5}{{}^{11}C_6\ 3^6} = \frac{1}{3}$

Sol.7 **A,B,C,D**

Coeffi of middle term in $(1 + x)^{2n}$ is $T_{\frac{2n}{2}+1} = T_{n+1}$

$T_{n+1} = {}^{2n}C_n x^n$

Coeff of $T_{n+1} = {}^{2n}C_n \Rightarrow$ (B)

${}^{2n}C_n = \frac{(2n)!}{n!\,n!} = \frac{2^n\ n!\,(1\cdot 3\cdot 5\ldots\ldots(2n-1))}{n!\ n!}$

$= \frac{1\cdot 3\cdot 5\ldots\ldots(2n-1)}{n!}\cdot 2^n \Rightarrow$ (A)

$= \frac{2\cdot 6\cdot 10\ldots\ldots 2(2n-3)\cdot 2(2n-1)}{n!}$

$= \frac{2\cdot 6\cdot 10\cdot 14\ldots\ldots(4n-6)(4n-2)}{1\cdot 2\cdot 3\cdot 4\ldots\ldots(n-1)n} \Rightarrow$ (D)

${}^{2n}C_n = \frac{2n\cdot(2n-1)\cdot(2n-2)\ldots\ldots(n+1)(n)!}{n(n-1)(n-2)\ldots\ldots 3\cdot 2\cdot 1\cdot n!}$

$\frac{(n+1)(n+2)(n+3)\ldots\ldots(2n-1)2n}{1\cdot 2\cdot 3\ldots\ldots(n-1)(n)} \Rightarrow$ (C)